

Volumen und Oberfläche eines Spats

Der Spat ist gegeben durch die Eckpunkte A, B, D und E.

$$A(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline \end{array})$$

$$B(\begin{array}{|c|c|c|} \hline -1 & 5 & 6 \\ \hline \end{array})$$

$$D(\begin{array}{|c|c|c|} \hline 8 & 2 & 1 \\ \hline \end{array})$$

$$E(\begin{array}{|c|c|c|} \hline -2 & 0 & 5 \\ \hline \end{array})$$

$$AB = \begin{array}{|c|} \hline -1 \\ \hline 5 \\ \hline 6 \\ \hline \end{array} \quad AD = \begin{array}{|c|} \hline 8 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} \quad AE = \begin{array}{|c|} \hline -2 \\ \hline 0 \\ \hline 5 \\ \hline \end{array}$$

$$c = b + AD \quad C(\begin{array}{|c|c|c|} \hline 7 & 7 & 7 \\ \hline \end{array})$$

$$f = b + AE \quad F(\begin{array}{|c|c|c|} \hline -3 & 5 & 11 \\ \hline \end{array})$$

$$g = c + AE \quad G(\begin{array}{|c|c|c|} \hline 5 & 7 & 12 \\ \hline \end{array})$$

$$h = d + AE \quad H(\begin{array}{|c|c|c|} \hline 6 & 2 & 6 \\ \hline \end{array})$$

Volumen:

$$V = \text{Betrag}(\det(AB \ AC \ AD)) = \begin{vmatrix} -1 & 8 & -2 \\ 5 & 2 & 0 \\ 6 & 1 & 5 \end{vmatrix} = \begin{vmatrix} -196 \end{vmatrix} = \boxed{196}$$

$$AB \times AD = \begin{array}{|c|} \hline -7 \\ \hline 49 \\ \hline -42 \\ \hline \end{array}$$

$$A_{ABCD} = \text{Betrag}(AB \times AD) = \boxed{64,915329}$$

$$AB \times AE = \begin{array}{|c|} \hline 25 \\ \hline -7 \\ \hline 10 \\ \hline \end{array}$$

$$A_{ABFE} = \text{Betrag}(AB \times AE) = \boxed{27,820855}$$

$$AD \times AE = \begin{array}{|c|} \hline 10 \\ \hline -42 \\ \hline 4 \\ \hline \end{array}$$

$$A_{ADHE} = \text{Betrag}(AD \times AE) = \boxed{43,358967}$$

Oberfläche:

$$O = 2 * (A_{ABCD} + A_{ABFE} + A_{ADHE}) = \boxed{272,1903}$$